MATH 1132 Honors
Activity 3 (Ch 11.2)

Name: ______________________

The Hidden Beauty of **The Harmonic Series**

GOAL: In this activity, we will take a closer look at the harmonic series:

$$\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \dots$$

In particular, we will explore two applications of the *divergence* property of this series.

BACKGROUND: As we have learned, the harmonic series $\sum_{n=1}^{\infty} \frac{1}{n}$ diverges (in particular, the sum is infinite). This can be a hard concept to wrap our minds around since the divergence of the series is very slow (as shown in the following table).

SUM	# of terms needed
1	1
2	4
4	31
6	227
8	1674
10	12367

That's unbelievable. You need to add up $12,367$ terms just to get a sum that exceeds 10! That means you need to add $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \cdots + \frac{1}{12367}$. If you want a sum of 100 you need to add on the order of 10^{44} terms. Ultimately, no matter what sum we want to achieve, we can always achieve it with enough terms of the harmonic series (it just takes a long time). The divergence of this series has several cool applications, as we will now see.

The LEANING TOWER of LIRE

The Leaning Tower of Lire is a puzzle that involves stacking blocks on the edge of a table. The goal is to take a collection of identical blocks/ objects, stack them one on top of the other on the edge of a table, and try to have the upper block hang as far as possible over the edge of the table. Can you create the stack so that the upper block is a full block length from the edge of the table? How about 2 full block lengths? 3? 10?

CHALLENGE #1: With two playing cards stacked on top of one another, how far past the edge of the table can your top card reach? (measure in card width's)

CHALLENGE #2: With three playing cards stacked on top of one another, how far past the edge of the table can your top card reach? (measure in card width's)

CHALLENGE #3: With a deck of cards, create a stack so that the upper card is as far past the edge of the table as possible. You may only use 1 card per level (i.e. no "counterweights"). Let's see which group can achieve the largest overhang with the smallest number of cards. (**Hint:** The Harmonic Series will apply here)

QUESTIONS:

[**A**] How does the *Harmonic Series* apply to this puzzle?

[**B**] In theory, it is actually possible to achieve an arbitrarily large overhang with a sufficient number of cards. Why? How many cards would we need in order to achieve an overhang that is 3 times the length of the card?

Gabriel's WEDDING CAKE

What if I told you that there existed a cake (a very large cake) that is possible to eat but impossible to frost? Such a cake exists and is called Gabriel's Wedding Cake! Here's how to make it:

- The wedding cake will be made by stacking cylindrical cakes on top of one another. There will be an infinite number of layers.
- Each layer (i.e. cylinder) will have the same height of 1 foot.
- The radius of the first layer will be 1 foot, the radius of the 2nd will be $\frac{1}{2}$ of a foot, the radius of the 3rd will be $\frac{1}{3}$ of a foot, etc. The radius of the kth layer will be $\frac{1}{k}$ feet.

QUESTIONS:

[**A**] Find a formula for the total volume of the cake in cubic feet (this is how much cake you'd have to eat). Use this formula to find the total volume.

[**B**] A standard serving of cakes is a 2"x2"x2" cube. How many guests will Gabriel's wedding cake serve?

[**C**] Find a formula for the total surface area of the cake in square feet (this is how much cake you'd have to frost).

[**D**] Why is this cake possible to eat but impossible to frost?